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Prediction of Resonant Frequencies of Rectangular, Circular and Triangular Microstrip Antennas using a Generalized RBF Neural Model

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Abstract—Microstrip antennas have proved to be the excellent radiators for many applications. It is so because of their numerous advantages such as light weight, low profile, conformable to planar and non-planar surface, low fabrication cost because of printed-circuit technology, integrability with other microwave integrated circuits (MICs) on the same substrate etc. Because of inherent characteristic of microstrip antennas to operate in the vicinity of resonant frequency, this resonant frequency needs to be calculated accurately. This paper presents a simple, accurate and fast approach based on radial basis function (RBF) neural networks for predicting the resonant frequencies of rectangular, circular and triangular microstrip antennas, simultaneously. The computed results are in very good agreement with their measured counterparts.

Index Terms— Resonant Frequencies, microstrip antennas, rectangular patch, circular patch, triangular patch, generalized approach and RBF neural networks.

1 INTRODUCTION

The conventional methods [1-3] like: transmission line model, cavity model, and full wave analysis are popularly used in analyzing and designing the microstrip antennas (MSAs). For the purpose, three stages are basically, involved in applying these methods on the electromagnetic (EM) problems. Firstly, the problem is formulated by creating a set of system equations to be solved. The geometry, describing the problem with necessary discretization into smaller elements, is created in the second stage and finally, these equations are solved using the method of choice. There are various circumstances like optimizing the problem geometry for optimum outputs, where repetitive computation of EM field is required. A minor alteration in the geometry requires a different discretization which itself is a time consuming exercise. Further, all these techniques have their own strong and weak points and require elaborate mathematics in applying on EM problems. Recently, the artificial neural networks (ANNs) have acquired tremendous utilization in analyzing and designing the MSAs [4-13]. It is so because the neural-models are computationally much more efficient than conventional models and require lesser time to model a circuit. A neural-model is trained off-line using few patterns generated through measurement, simulation and/or analytical model suitable for a problem. Once it is trained for a specified error then it returns the results for every infinitesimal changes in the input patterns within a fraction of a second and thus, completely bypasses the repetitive use of conventional models as the conventional models need re-discretization for every infinitesimal changes in the geometry which itself is a lingering and time-consuming exercise. Karaboga et. al. [5] have used a structure of two hidden layers with five neurons in each layer and a gradient descent with momentum backpropagation algorithm for training their neural model. Using this algorithm, they have calculated the resonant frequencies of electrically thin and thick rectangular MSAs with an average absolute error of 16.33 MHz. Nurhan Turker et. al. [6] have proposed multi-layered perceptron neural networks model of two hidden layers too. They have calculated the resonant frequencies of the rectangular MSAs with an average absolute error of 50 MHz. Ouchar et. al. [7] have used multi-layered perceptron artificial neural networks model with backpropagation training algorithm for calculating the resonant frequencies of the circular MSAs and the average absolute error in this model is calculated as 34.61MHz. Sagiroglu et. al. [8] have calculated the resonant frequencies of the circular MSAs using neural approach. They have used standard backpropagation algorithm with learning coefficient of 0.08 and the momentum coefficient of 0.10. The average absolute error in this model is calculated as 1.85 MHz. Sagiroglu and Guney [9] have used gradient-descent with momentum backpropagation algorithm for computing the resonant frequencies of equilateral triangular MSAs. Using this model, they have calculated the resonant frequencies of equilateral triangular MSAs with an average absolute error of 1.53 MHz. Thus the neural models [5-9] have been used for computing only single parameter i.e. the resonant frequencies of rectangular, circular and triangular microstrip antennas, respectively.

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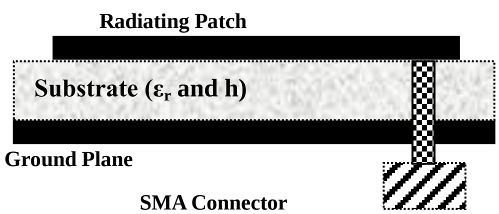
Recently the concept of using generalized neural models has been proposed for computing the resonant frequencies of rectangular, circular and triangular MSAs, simultaneously [10-13]. In these models the equivalent patch dimensions for the circular and triangular MSAs have been obtained by equating the patch areas of the circular and triangular MSAs to that of an equivalent rectangular MSA. Guney et. al. [10] have used two hidden layers with twelve and six neurons, respectively. Thus structural configuration of this neural model is complex and the calculated results by this model are also not in good agreement with their measured counterparts. Further, the Tabu search algorithm [11], ANFIS method [12] and CNFS method [13] have also been used for getting more accurate results but these approaches are also very complex. In present work, a very simple approach has been proposed for getting more accurate results than that of generalized models [10-13]. The proposed method based on radial basis function (RBF) neural networks is so simple that it can be trained from the measured, simulated and/or calculated results but to understand the novelty of the proposed work initially, it has been decided to use the measured results [14-26] as training and testing patterns. Once it is tested and validated successfully, it can be generalized on measured, simulated and/or calculated patterns.

2. RBF NEURAL NETWORKS

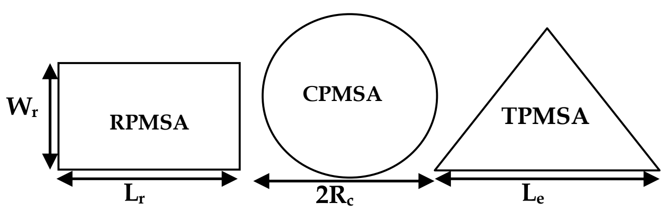
The radial basis function neural networks (RBFNN) and the multi-layered perceptron neural networks (MLPNN) are the two popular models used in different applications of microwave domain [5-13]. A RBF neural networks consists of three layered feed-forward neural networks with entirely different tasks. The input layer is made-up of source nodes which connect the network to the outside environment. A multi-variate Gaussian non-linear transformation is used as an activation function in the hidden layer and the output layer supplies the response of the network and does not have any activation function. During training process, a RBFNN model is developed by learning from the available patterns. The aim of training process is to minimize the error between actual output and calculated output from the RBFNN model. As far as training is concerned, RBFNN is much faster than MLPNN. It is so because the training process in RBFNN has two stages and both the stages are made more efficient by using appropriate training algorithm. That is the reason of using RBFNN instead of MLPNN in many applications. Once the model is trained for a specified error, then it returns the results for every infinitesimal changes in the applied input patterns within a fraction of a second. In general three common steps are used in applying neural networks for instantly predicting the desired performance parameter; resonant frequency of rectangular, circular or triangular MSAs. Firstly, the training and testing patterns are generated and the structural configuration of hidden layer neurons is selected for training in the second step. And finally, training algorithm is applied on RBFNN model in the third step. The detailed description of each step involved is being discussed in the subsequent sub-sections below:

2.1 Generation of Patterns

A microstrip antenna, in its simplest configuration, consists of a radiating conductive patch on one side of a dielectric substrate of relative permittivity 'ε_r' and of thickness 'h' having a ground plane on the other side [1-3]. The side-view of a microstrip antenna and different radiating patches used in present work are shown in Fig. 1



(a) Side-view



(b) Different Radiating patches

Fig. 1: Microstrip Patch Antennas

In Fig.1, RPMSA corresponds to a rectangular patch of physical dimensions 'W_r' and 'L_r', CPMSA to a circular patch of radius 'R_c' and TPMSA to an equilateral triangular patch of side-length 'L_e'. It is clear from the literature [14-26] that the resonant frequency of a microstrip antenna is the function of physical dimension(s), relative permittivity, dielectric thickness and mode of propagation. Total 81 data patterns (46 for RPMSA, 20 for CPMSA and 15 for TPMSA) have been arranged from the literature [14-17], [18-24] and [25-26], respectively. It is clear from Table 1 that the calculating parameter of a radiating patch is the function of five input parameters (x_1, x_2, x_3, x_4, and x_5) of the patch in three different cases. To distinguish them, an arbitrary parameter, 'x_6', is also included in 5-dimensional input patterns where x_6 = 1, 2 and 3 corresponds to the resonant frequency of RPMSAs, resonant frequency of CPMSAs and resonant frequency of TPMSAs, respectively.

Table 1: Input-Output Patterns for RBFNN Training

CASE I: Resonant Frequency of RPMSA (x_6=1)

Patch Parameters	ANN Inputs	ANN Output (GHz)
Width of the Patch (cm)	x_1	Resonant Frequency
Length of the Patch (cm)	x_2	of RPMSA
Dielectric Thickness (cm)	x_3	(Total patterns=46)
Dielectric Constant (ε_r)	x_4	**[14-17]**
Mode of Propagation (m&n)	x_5	

CASE II: Resonant Frequency of CPMSA (x_6=2)

Patch Parameters	ANN Inputs	ANN Output (GHz)
Radius of the Patch (cm)	x_1	Resonant Frequency
Dielectric Thickness (cm)	x_2	of CPMSA
Dielectric Constant (ε_r)	x_3	(Total patterns=20)
Mode of Propagation (m)	x_4	**[18-24]**
Mode of Propagation (n)	x_5	

CASE III: Resonant Frequency of TPMSA (x_6=3)

Patch Parameters	ANN Inputs	ANN Output (GHz)
Side-Length of Patch (cm)	x_1	Resonant Frequency
Dielectric Thickness (cm)	x_2	of TPMSA
Dielectric Constant (ε_r)	x_3	(Total patterns=15)
Mode of Propagation (m)	x_4	**[25-26]**
Mode of Propagation (n)	x_5	

2.2 Proposed ANN Structure and Algorithms

Selecting the structural configuration of RBF neural networks and neurons in the hidden layer is the prime requirement before applying training algorithm on the neural networks [4]. The training performance of the neural networks is observed by varying the number of neurons in the hidden layer and finally, it is optimized with sixteen neurons for the best performance. Further, the proposed RBF neural networks is trained with seven different algorithms [27-29]; BFGS quasi-Newton backpropagation (BFG), Bayesian regulation backpropagation (BR), scaled conjugate gradient backpropagation (SCG), Powell-Beale conjugate gradient backpropagation (CGP), conjugate gradient backpropagation with Fletcher-Peeves (CGP), one step secant backpropagation (OSS), and Levenberg-

Marquardt backpropagation (LM) and only the LM backpropagation [29] is proved to be the fastest converging training algorithm and produced the results with least error as can be confirmed from Table 2. Total 81 measured samples are divided into 66 training samples (37 for resonant frequencies of RPMSA, 17 for CPMSA and 12 for TPMSA) whereas remaining 15 samples are validated during the testing of the RBF neural networks. All initial weights and bias values are selected randomly and rounded-off between -1.0 and +1.0. The mean square error (MSE), learning rate, momentum coefficient and spread value is taken as: 5×10^{-7}, 0.1, 0.5 and 0.5, respectively and epochs required for getting a MSE of 5×10^{-7} is only 958. After getting training successfully, one can predict the resonant frequency of any arbitrary patch; rectangular, circular or equilateral triangular for the given arbitrary set of input parameters within their specified ranges. This is shown in Fig. 2.

Table 2: Comparison of Error vs. Training Algorithm

Training Algorithm [27-29]	Average Absolute Error in analysis			Iteration Required
	RPMSAs (MHz)	CPMSAs (MHz)	ETMSAs (MHz)	
BFG	2.244	17.155	9.680	2241
BR	6.337	16.370	7.687	2863
SCG	4.374	14.265	13.093	2312
CGP	4.683	15.260	19.487	2101
CGF	3.754	11.985	19.273	2161
OSS	3.085	11.575	20.180	2160
LM	1.922	1.145	1.553	958

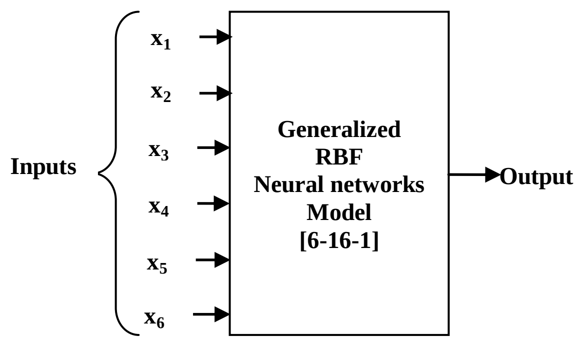


Fig. 2: Proposed GRBFNN

3. CALCULATED RESULTS AND DISCUSSION

The resonant frequencies calculated using the generalized neural model for rectangular, circular and triangular MSAs are listed in Table 3, Table 4 and Table 5, respectively. For comparison, the neural results obtained by Karaboga et al [5], Turker et. al. [6] and Guney and Sarikaya [12-13] are given in Table 3, by Oucher et. al. [7], Guney et. al. [8] and Guney and Sarikaya [12-13] in Table 4**,** by Guney and Sarikaya [12-13], Guney et. al. [10] and Sagiroglu and Guney [9] in Table 5 for resonant frequencies of rectangular, circular and triangular MSAs, respectively. Table 3 shows that in the models [5], [6], [12], and [13] the average absolute error for resonant frequencies of rectangular MSAs is calculated as 16.33MHz, 50.0MHz, 6.1697MHz and 16.8818MHz whereas in the present model it is only 1.922MHz. In case of circular MSAs, the present method is having the average absolute error of 1.145MHz whereas in the models [12], [13], [10], [8] and [7], it is calculated as 4.60MHz, 5.84MHz, 23.09MHz, 0.550MHz and 4.60MHz, respectively**.** For triangular MSAs, the models [12], [13], [10] and [9] are having the average absolute error as 1.773MHz, 1.873MHz, 18.127MHz and 1.533MHz, respectively whereas in the present method, it is only 1.553MHz.

Further illustrating the proposed work, the average absolute errors between the measured and calculated results are also compared in Tables 4-6. It is clear from these three tables that the results in the proposed method are closer with their measured counterparts as compared to the previous ANN results [5, 6, 7, 8, 9, 10, 11, 12 and 13] in all three different computing parameters. A very good convergence between the measured and calculated results supports the validity of the generalized neural method. It is clear from Table 4 that the present approach is less accurate than the approach proposed by Sagiroglu et. al. [8] in case of circular MSAs but the model [8] is calculating only one parameter whereas the present model is calculating the resonant frequencies in three different MSAs.

4. CONCLUSION

The generalized neural method based on radial basis function has been presented to accurately and simultaneously computing the resonant frequencies of the rectangular, circular, and triangular MSAs as such an accurate and simple approach is rarely available in the literature. The RBF neural model has been trained with seven different training algorithms and only Levenberg-Marquardt training algorithm is proved to be the most accurate. The results of the proposed method are in very good agreement with their measured counterparts, and has the better accuracy with respect to the neural models proposed in the literature [5-13]. The main advantage of the proposed method is that the single hidden layer structure with only 16 neurons is used for calculating the three different parameters of three different microstrip antennas. The proposed approach offers an accurate and efficient single alternative to the independent neural models [5-9]. The quick, accurate and efficient computation feature of the proposed method recommends developing some embedded neural simulators on microcontrollers, DSP processors or on FPGA platforms that would open some novel paradigms in the microwave community for effectively utilizing the artificial neural networks on some sort of hardware. Further, the approach can also be included in antenna computer-aided designs because of computing three different parameters of three different microstrip antennas accurately and simultaneously.

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tion, IEE Proceedings, vol. 146, no. 2, pp. 155-159, 1999.

Table 3: Calculated Results for Rectangular MSAs and Comparison with Previous ANN Results

W_r (cm)	L_r (cm)	h (cm)	ε_r	Mode	Measured *f_r(GHz)	Calculated #f_{rn}(GHz)	Previous ANN Results			
							Ref. [5]	Ref. [6]	Ref. [12]	Ref. [13]
5.7000	3.8000	0.3175	2.3300	TM_{10}	2.3100	2.3107	2.3090	2.3109	-	-
4.5500	3.0500	0.3175	2.3300	TM_{10}	2.8900	2.8687	2.8900	2.8881	-	-
2.9500	1.9500	0.3175	2.3300	TM_{10}	4.2400	4.2413	4.2240	4.2061	-	-
1.9500	1.3000	0.3175	2.3300	TM_{10}	5.8400	5.8400	5.8410	5.8890	-	-
1.7000	1.1000	0.3175	2.3300	TM_{10}	6.8000	6.8003●	6.8320	6.6959	-	-
1.4000	0.9000	0.3175	2.3300	TM_{10}	7.7000	7.6909	7.7040	7.7950	-	-
1.2000	0.8000	0.3175	2.3300	TM_{10}	8.2700	8.2654	8.2700	8.3661	-	-
1.0500	0.7000	0.3175	2.3300	TM_{10}	9.1400	9.1381	9.1460	9.0720	-	-
1.7000	1.1000	0.9525	2.3300	TM_{10}	4.7300	4.7310	4.7280	4.6867	-	-
1.7000	1.1000	0.1524	2.3300	TM_{10}	7.8700	7.8702	7.8390	-	-	-
4.1000	4.1400	0.1524	2.5000	TM_{10}	2.2280	2.2279	2.2260	-	-	-
6.8580	4.1400	0.1524	2.5000	TM_{10}	2.2000	2.2010●	2.1900	-	-	-
10.8000	4.1400	0.1524	2.5000	TM_{10}	2.1810	2.1809	2.1810	-	-	-
0.8500	1.2900	0.0170	2.2200	TM_{10}	7.7400	7.7383	7.7360	-	7.7438	7.7400
0.7900	1.1850	0.0170	2.2200	TM_{10}	8.4500	8.4496●	8.4140	-	8.4555	8.2640
2.0000	2.5000	0.0790	2.2200	TM_{10}	3.9700	3.9681	3.9660	-	3.9711	3.9700
1.0630	1.1830	0.0790	2.5500	TM_{10}	7.7300	7.7271	7.7250	-	7.7266	7.7300
0.9100	1.0000	0.1270	10.2000	TM_{10}	4.6000	4.6003	4.6000	-	4.5988	4.6000
1.7200	1.8600	0.1570	2.3300	TM_{10}	5.0600	5.0599	5.0580	5.0259	5.0575	5.0599
1.8100	1.9600	0.1570	2.3300	TM_{10}	4.8050	4.8049●	4.8280	-	4.8426	4.8324
1.2700	1.3500	0.1630	2.5500	TM_{10}	6.5600	6.5597	6.5420	-	6.5598	6.5599
1.5000	1.6210	0.1630	2.5500	TM_{10}	5.6000	5.5991	5.5810	-	5.5947	5.6003
1.3370	1.4120	0.2000	2.5500	TM_{10}	6.2000	6.2001●	6.1890	-	6.1813	6.1936
1.1200	1.2000	0.2420	2.5500	TM_{10}	7.0500	7.0493	7.0230	-	7.0489	7.0501
1.4030	1.4850	0.2520	2.5500	TM_{10}	5.8000	5.7948	5.8010	-	5.8007	5.7994
1.5300	1.6300	0.3000	2.5000	TM_{10}	5.2700	5.2681	5.2660	-	5.2776	5.2702
0.9050	1.0180	0.3000	2.5000	TM_{10}	7.9900	7.9889	7.9670	-	7.9916	7.9900
1.1700	1.2800	0.3000	2.5000	TM_{10}	6.5700	6.5680	6.5540	-	6.5701	6.5702
1.3750	1.5800	0.4760	2.5500	TM_{10}	5.1000	5.0968●	5.1570	-	5.0978	4.8378
0.7760	1.0800	0.3300	2.5500	TM_{10}	8.0000	7.9962	7.9900	-	7.9980	8.0000
0.7900	1.2550	0.4000	2.5500	TM_{10}	7.1340	7.1339	7.1070	7.0603	7.1349	7.1339
0.9870	1.4500	0.4500	2.5500	TM_{10}	6.0700	6.0684	6.0670	6.0940	6.0726	6.0698
1.0000	1.5200	0.4760	2.5500	TM_{10}	5.8200	5.8181●	5.8470	5.8600	5.8632	5.8323
0.8140	1.4400	0.4760	2.5500	TM_{10}	6.3800	6.3750	6.3920	6.4234	6.3803	6.3802
0.7900	1.6200	0.5500	2.5500	TM_{10}	5.9900	5.9851	5.9500	5.9439	5.9900	5.9899
1.2000	1.9700	0.6260	2.5500	TM_{10}	4.6600	4.6600	4.6320	-	4.6592	4.6600
0.7830	2.3000	0.8540	2.5500	TM_{10}	4.6000	4.6001	4.6020	-	4.6063	4.6000
1.2560	2.7560	0.9520	2.5500	TM_{10}	3.5800	3.5805●	3.5100	-	3.6005	3.5428
0.9740	2.6200	0.9520	2.5500	TM_{10}	3.9800	3.9802	3.9540	-	3.9721	3.9796
1.0200	2.6400	0.9520	2.5500	TM_{10}	3.9000	3.9031	3.8820	-	3.9070	3.9003
0.8830	2.6760	1.0000	2.5500	TM_{10}	3.9800	3.9812	3.9780	-	3.9845	3.9801
0.7770	2.8350	1.1000	2.5500	TM_{10}	3.9000	3.9002	3.9820	-	3.8947	3.9000
0.9200	3.1300	1.2000	2.5500	TM_{10}	3.4700	3.4702	3.4600	-	3.4725	3.4699
1.0300	3.3800	1.2810	2.5500	TM_{10}	3.2000	3.2002●	3.1870	-	3.1949	3.2224
1.2650	3.5000	1.2810	2.5500	TM_{10}	2.9800	2.9809	2.9630	-	2.9794	2.9800
1.0800	3.4000	1.2810	2.5500	TM_{10}	3.1500	3.1501	3.1410	-	3.1485	3.1501
Average Absolute Error (MHz)					→	**1.9220**	16.330	50.000	6.1697	16.882

*f_r→Measured Results [21-24], #f_{rn}→Calculated GRBFNN Results and ●→Testing Results.

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Table 4: Calculated Results for Circular MSAs and Comparison with Previous ANN Results

Input Parameters				Measured	Calculated	Previous ANN Results				
R_c (cm)	h(cm)	ε_r	Mode	*f_c (GHz)	#f_{cn} (GHz)	Ref. [12]	Ref. [13]	Ref. [10]	Ref. [8]	Ref. [7]
6.80000	0.08000	2.32000	TM_{11}	0. 8350	0.8351	0.8346	0.8356	0.8229	0.835	0.8351
6.80000	0.15900	2.32000	TM_{11}	0.8290	0.8289●	0.8233	0.8241	0.8202	0.828	0.8301
6.80000	0.31800	2.32000	TM_{11}	0.8150	0.8152	0.8158	0.8166	0.8145	0.815	0.8144
5.00000	0.15900	2.32000	TM_{11}	1.1280	1.1283	1.1280	1.1282	1.1081	1.128	1.1284
3.80000	0.15240	2.49000	TM_{11}	1.4430	1.4428	1.4444	1.4447	1.4404	1.443	1.4443
4.85000	0.31800	2.52000	TM_{11}	1.0990	1.0991	1.0977	1.0985	1.1096	1.099	1.0986
3.49300	0.15880	2.50000	TM_{11}	1.5700	1.5685	1.5703	1.5728	1.5655	1.570	1.5678
1.27000	0.07940	2.59000	TM_{11}	4.0700	4.0709	4.0707	4.0703	4.1443	4.070	4.0703
3.49300	0.31750	2.50000	TM_{11}	1.5100	1.5102	1.5100	1.5115	1.5617	1.510	1.5117
4.95000	0.23500	4.55000	TM_{11}	0.8250	0.8253	0.8248	0.8257	0.8824	0.825	0.8250
3.97500	0.23500	4.55000	TM_{11}	1.0300	1.0307	1.0305	1.0323	1.0280	1.030	1.0313
2.99000	0.23500	4.55000	TM_{11}	1.3600	1.3591	1.3603	1.3593	1.3126	1.361	1.3592
2.00000	0.23500	4.55000	TM_{11}	2.0030	2.0030●	2.0348	2.0321	1.9791	2.003	2.0048
1.04000	0.23500	4.55000	TM_{11}	3.7500	3.7481	3.7495	3.7501	3.7322	3.750	3.4980
0.77000	0.23500	4.55000	TM_{11}	4.9450	4.9425	4.9429	4.9450	4.9655	4.945	4.9458
1.15000	0.15875	2.65000	TM_{11}	4.4250	4.4257	4.4242	4.4243	4.4287	4.425	4.4209
1.07000	0.15875	2.65000	TM_{11}	4.7230	4.7131	4.7204	4.7226	4.7123	4.723	4.7292
0.96000	0.15875	2.65000	TM_{11}	5.2240	5.2236	5.2253	5.2247	5.1980	5.224	5.2249
0.74000	0.15875	2.65000	TM_{11}	6.6340	6.6322	6.6718	6.7014	6.6625	6.634	6.2523
0.82000	0.15875	2.65000	TM_{11}	6.0740	6.0738●	6.0715	6.0734	6.0450	6.074	6.1084
Average Absolute Error (MHz) →					**1.1450**	4.6000	5.8400	23.100	0.550	4.6000

*f_c→Measured Results [25-31], #f_{cn}→ Calculated GRBFNN Results and ●→Testing Results.

Table 5: Calculated Results for Triangular MSAs and Comparison with Previous ANN Results

Input Parameters				Measured	Calculated	Previous ANN Models			
L_e(cm)	h(cm)	ε_r	Mode	*f_e(GHz)	#f_{en}(GHz)	Ref. [12]	Ref. [13]	Ref. [10]	Ref. [9]
4.1000	0.0700	10.5000	TM_{10}	1.5190	1.5185●	1.5175	1.5191	1.5270	1.5260
4.1000	0.0700	10.5000	TM_{11}	2.6370	2.6365	2.6358	2.6340	2.6235	2.6370
4.1000	0.0700	10.5000	TM_{20}	2.9950	2.9957	2.9968	2.9957	2.9833	2.9950
4.1000	0.0700	10.5000	TM_{21}	3.9730	3.9741	3.9730	3.9732	3.9921	3.9730
4.1000	0.0700	10.5000	TM_{30}	4.4390	4.4392	4.4386	4.4387	4.4245	4.4390
8.7000	0.0780	2.3200	TM_{10}	1.4890	1.4885●	1.4888	1.4899	1.5037	1.4780
8.7000	0.0780	2.3200	TM_{11}	2.5960	2.5943	2.5958	2.5954	2.6006	2.5960
8.7000	0.0780	2.3200	TM_{20}	2.9690	2.9704	2.9695	2.9679	2.9866	2.9690
8.7000	0.0780	2.3200	TM_{21}	3.9680	3.9671	3.9774	3.9776	3.9456	3.9680
8.7000	0.0780	2.3200	TM_{30}	4.4430	4.4441	4.4423	4.4424	4.4402	4.4430
10.0000	0.1590	2.3200	TM_{10}	1.2800	1.2838	1.2804	1.2787	1.2577	1.2800
10.0000	0.1590	2.3200	TM_{11}	2.2420	2.2425	2.2424	2.2429	2.2241	2.2420
10.0000	0.1590	2.3200	TM_{20}	2.5500	2.5409	2.5494	2.5506	2.5009	2.5500

10.0000	0.1590	2.3200	TM_{21}	3.4000	3.4013	3.3982	3.4001	3.4165	3.4000
10.0000	0.1590	2.3200	TM_{30}	3.8240	3.8247●	3.8315	3.8321	3.8612	3.8290
Average Absolute Error (MHz) ⟶					**1.5530**	1.7730	1.8730	18.127	1.5330

*f_e→Measured Results [32-33], #f_{en}→ Present Method Results ●→Testing Results.

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